TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 2018

Section 1 Calculator Free

Systems of Equations, Vector Calculus

STUDENT'S NAME ______________________________

DATE: Friday 18 May **TIME:** 20 minutes **MARKS**: 19

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Three planes are defined by the following equations:

$3x + 2y - z = 19$, $\underset{\sim}{\mathbf{r}} \cdot \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} = 4$ and $2x + 4y - 5z = 32$. Determine the coordinates of the unique point of intersection of the three planes, using techniques of elimination.

2. (8 marks)

Given $\underset{\sim}{\mathbf{r}}(t) = \left(3\sin\frac{t}{2}\right)\underset{\sim}{\mathbf{i}} + \left(2\cos\frac{t}{2}\right)\underset{\sim}{\mathbf{j}}$, where $\underset{\sim}{\mathbf{r}}(t)$ is the position vector of a particle at time t,

(a) Determine the cartesian equation of the path of the particle stating its shape. (3 marks)

(b) Determine $\underset{\sim}{\mathbf{v}}(t)$ and $\underset{\sim}{\mathbf{a}}(t)$ (2 marks)

(c) Show that $\underset{\sim}{\mathbf{v}}(t)\cdot\underset{\sim}{\mathbf{a}}(t) = -\frac{5}{16}\sin t$ (3 marks)

3. (7 marks)

Consider the following system of equations. Note: k is a constant.

$$\begin{aligned} x - 2y + 3z &= 1 \\ x + ky + 2z &= 2 \\ -2x + k^2y - 4z &= 3k - 4 \end{aligned}$$

(a) State the value(s) of k for which the system has an infinite number of solutions and give a geometric interpretation. (4 marks)

(b) State the value(s) of k for which the system has no solution. (1 mark)

(c) For what value(s) of k does the system have a unique solution? (2 marks)

TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 3 2018

Section 2 Calculator Assumed

Systems of Equations. Vector Calculus

STUDENT'S NAME ____

DATE: Friday 18 May **TIME:** 35 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (13 marks)

The velocity vector $\underset{\sim}{\mathbf{v}}(t)\ ms^{-1}$ of a particle is given by $\underset{\sim}{\mathbf{v}}(t) = \left(\frac{3\pi}{4}\cos\frac{\pi t}{4}\right)\underset{\sim}{\mathbf{i}} - \left(\frac{3\pi}{4}\sin\frac{\pi t}{4}\right)\underset{\sim}{\mathbf{j}}$.

The position vector of the particle at time $t = 4$ is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(a) Determine, for any time t

(i) the displacement vector $\underset{\sim}{\mathbf{r}}(t)$ (2 marks)

(ii) the speed $|\underset{\sim}{\mathbf{v}}(t)|$ (1 mark)

(iii) the acceleration $\underset{\sim}{\mathbf{a}}(t)$ (1 mark)

(b) Describe the motion of the particle indicating the direction. (3 marks)

(c) Evaluate and interpret each of the following integrals.

(i) $\displaystyle\int_0^6 \underset{\sim}{\mathbf{v}}(t)\,dt$ (2 marks)

(ii) $\displaystyle\int_0^6 |\underset{\sim}{\mathbf{v}}(t)|\,dt$ (2 marks)

(iii) $\displaystyle\left|\int_0^6 \underset{\sim}{\mathbf{v}}(t)\,dt\right|$ (2 marks)

5. (11 marks)

A point Q moving in the x-y plane has position vector $\underset{\sim}{\mathbf{r}}(t) = \begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix}$. At $t = 0$ an insect crawls from the origin towards Q so that its position vector at time t is $\underset{\sim}{\boldsymbol{R}}(t) = \underset{\sim}{\mathbf{r}}(t) \times \sin t$, until it reaches Q, where it rests until $t = \dfrac{9\pi}{4}$ minutes.

(a) Determine the position vector of Q when $t = 0$ (1 mark)

(b) How long does it take for the insect to first reach Q? (2 marks)

(c) Show that $\underset{\sim}{\boldsymbol{R}}(t) = \begin{pmatrix} \dfrac{\sin 2t}{2} \\ 1 - \cos 2t \end{pmatrix}$ (2 marks)

(d) Determine the cartesian equation for the path of the insect before it reaches Q. (2 marks)

(e) Sketch the path of the insect indicating its direction. (2 marks)

(f) Determine the relationship between the velocity and acceleration of the insect at $t = \frac{\pi}{4}$

(2 marks)

6. (10 marks)

A tennis ball is hit with an initial velocity of $\begin{pmatrix} 26 \cdot 5 \\ 2 \cdot 7 \end{pmatrix} ms^{-1}$ at a height of 60 cm above the ground and 6.4 m from the net. The net is 0.9 m high and the opponents half of the court is twelve metres in length.

(a) Determine the velocity vector and the position vector of the ball in terms of t (time) if the acceleration acting on the ball is given by $\boldsymbol{a} = \begin{pmatrix} 0 \\ -9 \cdot 8 \end{pmatrix} ms^{-2}$. (4 marks)

(b) Will the ball clear the net and if so by how much? (3 marks)

(c) Will the ball land inside the opponent's half? Justify. (3 marks)